# Methods of Point Estimation. Method of Moments

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## Motivation

Practical situation: we know that random data is drawn from a parametric model (distribution), whose parameters we do not know.

For example, in an election between two candidates, data will be drawn from a Bernoulli(p) distribution with unknown parameter p. Use the data to estimate the value of the parameter p, as p predicts the result of the election.

- Before... If the distribution (model) and its parameters are known then we can calculate the probability of data.
- ► *Now with Stat. Inference...* Estimate the probability of parameters given a parametric model and observed data drawn from it.

I know that data follows the Normal distributions but don't know the values of the parameters  $\mu$  and  $\sigma^2$ . However, data from a random sample is available to draw inference about  $\mu$  and  $\sigma^2$ .

## Methods of Point Estimation

- How to estimate a parameter?
- Estimating a parameter with its sample analogue is usually reasonable
- Still need a more methodical way of estimating parameters
- Method of Moments (MOM) is the oldest method of finding point estimators
- MOM is simple and often doesn't give best estimates
- Method of maximum likelihood (ML or MLE)
- MLEs have better efficiency properties than MOM estimates. But moment estimators are sometimes easier to compute
- Both ML and MOM can produce unbiased point estimators
- ► Bayesian Estimation of Parameters: prior information + sample results

## Method of Moments

Idea: equate the first k population moments, which are defined in terms of expected values, to the corresponding k sample moments. Solve the system of equations.

Let  $X_1, X_2, ..., X_n$  be a random sample from the probability distribution (discrete or continuous). The *kth* population moment (or distribution moment) is  $E(X^k), k = 1, 2, ...$  The corresponding *kth* sample moment is

$$\frac{1}{n}\sum_{i=1}^{n}X_{i}^{k}, \quad k=1,2,...$$

Example: the first population moment is  $E(X) = \mu$ , and the first sample moment (k = 1) is  $\overline{X}$ . Thus, by equating the population and sample moments, we find that  $\hat{\mu} = \overline{X}$ . The sample mean is the moment estimator of the population mean.

### Exponential Distribution Moment Estimator

Let  $X_1, X_2, ..., X_n$  be a random sample from the *Exponential*( $\lambda$ ) distribution. The question: which exponential distribution?!

- need to estimate one parameter  $\lambda$ , so k = 1
- MOM: equate  $E(X) = \overline{X}$  (population mean = sample mean)

$$egin{aligned} \mathsf{E}(X) &= 1/\lambda = ar{X} \ ar{X} &= rac{1}{\lambda} \ \hat{\lambda} &= rac{1}{ar{X}} \end{aligned}$$

is the moment estimator  $\lambda$ .

Suppose that the time to failure of an electronic module is exponentially distributed. Eight units are randomly selected and tested, resulting in the following failure time (in hours): 11.96, 5.03, 67.40, 16.07, 31.50, 7.73, 11.10, 22.38. The moment estimate of  $\lambda$  is

$$\hat{\lambda} = \frac{1}{\bar{x}} = \frac{1}{21.65} = 0.0462$$

#### Normal Distribution Moment Estimators

Let  $X_1, X_2, ..., X_n$  be a random sample from the *Normal*( $\mu, \sigma^2$ ) distribution. For the normal distribution,  $E(X) = \mu$  and  $E(X^2) = \mu^2 + \sigma^2$ .

- need to estimate two parameters, so k = 2
- ► MOM: equate

$$E(X) = \bar{X}, \qquad E(X^2) = \frac{1}{n} \sum_{i=1}^n X_i^2$$
  
 $\mu = \bar{X}, \qquad \mu^2 + \sigma^2 = \frac{1}{n} \sum_{i=1}^n X_i^2$ 

Solve

$$\hat{\mu}=ar{X}$$

and

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^n X_i^2 - n(\sum_{i=1}^n X_i)^2}{n} = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n}$$

Notice that the moment estimator of  $\sigma^2$  is not an unbiased estimator.