Midterm Review

Stat 345 - Spring 2020

Problem 1

Let X be a random variable with the following probability mass function:

$$p(x) = cx^2, \quad x = 0, 1, 4$$

and $p(x) = 0$ otherwise

a) What value of c makes this a valid probability mass function?

b) Find $E(2\sqrt{X})$.

Problem 2

Let X be a random variable with probability density function

$$f(x) = 1 - \frac{x}{2}, \quad 0 < x < 2$$

and $f(x) = 0$ otherwise

a) What is the cumulative distribution function of X?

b) Find $P(0.6 \leq X < 1)$.

c) Determine expected value of X.

Problem 3

a) Two married couples have bought 4 seats in a row for a performance of a musical comedy. In how many ways can they be seated if each couple is to sit together?

b) A woman's wardrobe consists of 3 jackets, 5 dresses, and 2 pairs of shoes. In how many ways can she select an outfit?

c) What is the probability of a single pair (for example, two Kings and three non-Kings) in the standard poker game?

Problem 4

Suppose that 5% of men and 0.25% of women are color-blind. A person is choosen at random and that person is color-blind. What is the probability that the person is male? (Assume males and females to be in equal numbers).

Problem 5

Let's say that 60% of all IT startups report that they generate a profit in their first year. If a sample of 100 new IT startups is selected,

a) find the probability that exactly 55 will generate a profit in their first year.

b) What is the probability that more than 70 will generate a profit in their first year?

Problem 6

In a clinical study, volunteers are tested for a gene that has been found to increase the risk for a disease. The probability that a person carries the gene is 0.1.

a) What is the probability that four or more people need to be tested to detect one person with the gene?

b) What is the expected number of people to test to detect two people with the gene?

Problem 7

The number of houses sold by an estate agent follows a Poisson distribution with a mean 2 per month. Find the probability that during a certain month the estate agent will sell more than 2 houses.

Problem 8

A driver's reaction time to visual stimulus is normally distributed with a mean of 0.4 seconds and variance of 0.0025 seconds. What is the probability that a reaction requires between 0.5 and 0.6 seconds?

Problem 9

The length of stay at a specific emergency department in a hospital in Phoenix, Arizona had a mean of 4 hours. Assume that the length of stay is exponentially distributed.

a) What length of stay is exceeded by 30% of the visits?

b) Given that a person has already waited two hours, what is the probability that this person will wait for another hour.