Distribution functions

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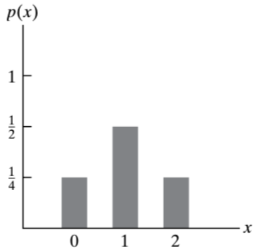
Probability mass function (pmf)

For a discrete random variable X with possible real values of x, a probability mass function is a function such that

- $p(x) \geq 0$
- $\sum_{\text{all } x} p(x) = 1$
- $p(x) = P(X = x)$

$$p(0) = \frac{1}{4} \qquad p(1) = \frac{1}{2} \qquad p(2) = \frac{1}{4}$$



Probability mass function. Example

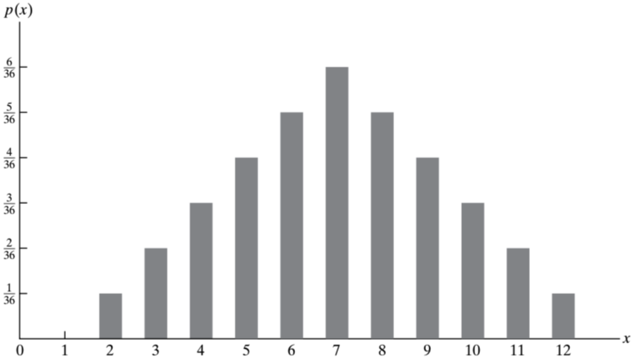


Figure 1: The probability mass function $p(x) = P(X = x)$ of the random variable X representing the sum when two dice are rolled

Cumulative distribution function (cdf)

With every random variable X, we associate a function called the cumulative distribution function (cdf) of X. For a random variable X, the function F (cdf) defined for all real values of x by

$$F(x) = P(X \leq x) = \sum_{i=1}^{x} P(X = i) = \sum_{i=1}^{x} p(i)$$

- The distribution function specifies that the probability that the random variable is less than or equal to x
- $0 \leq F(x) \leq 1$
- If $x \leq y$, then $F(x) \leq F(y)$, F is a non-decreasing function
- $P(X = x)$ can be determined from the jump at the value x

$$P(X = x) = P(X \leq x) - P(X < x) = F(x) - \lim_{y \uparrow x} F(y)$$

$y \uparrow x$ means y approaches x from below

Cumulative distribution function. Example

Determine the probability mass function (pmf) of X from the following cdf:

$$F(x) = \begin{cases} 0 & x < -2 \\ 0.2 & -2 \le x < 0 \\ 0.7 & 0 \le x < 2 \\ 1 & 2 \le x \end{cases}$$

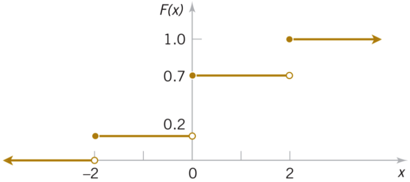


Figure 2: Cumulative distribution function

The only points that receive nonzero probability are -2, 0, and 2. The pmf at each point is the jump in the cumulative distribution function at the point. Therefore,

$$p(-2) = 0.2 - 0 = 0.2, p(0) = 0.7 - 0.2 = 0.5, p(2) = 1.0 - 0.7 = 0.3$$

$$p(2) = P(X = 2) = F(2) - \lim_{y \uparrow 2} F(y) = 1.0 - 0.7 = 0.3$$

Example

There is a chance that a bit transmitted through a digital transmission channel is received in error. Let X equal the number of bits in error in the next four bits transmitted. The possible values for X are $\{0, 1, 2, 3, 4\}$. Suppose that the probabilities are $P(X = 0) = 0.6561, P(X = 1) = 0.2916, P(X = 2) = 0.0486, P(X = 3) = 0.0036, P(X = 4) = 0.0001$. The probability distribution of X is specified by the possible values along with the probability of each. Find the probability that three or fewer bits are in error. Also find $P(1 < X \leq 3)$.

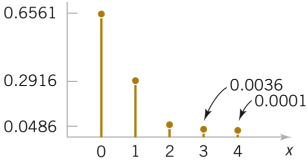


Figure 3: Probability distribution for bits in error

Example

The probability mass function of a random variable X is given by $p(i) = c\lambda^i/i!$, $i = 0, 1, 2, \ldots$, where λ is some positive value. Find $p(0)$ and $P(X > 2)$. Note that $e^x = \sum_{i=0}^{\infty} x^i/i!$