Poisson Distribution

Anastasiia Kim

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Poisson distribution

A random variable X that takes on one of the values $0, 1, 2, ...$ is said to be a Poisson random variable with parameter λ if, for some $\lambda > 0$,

$$p(x) = P(X = x) = e^{-\lambda} \frac{\lambda^x}{x!} \qquad x = 0, 1, 2, ...$$

with $E(X) = \lambda$ and $Var(X) = \lambda$

Poisson distribution

If n independent trials, each of which results in a success with probability p, are performed, then, when n is large and p is small enough to make np moderate, the number of successes occurring is approximately a Poisson random variable with parameter $\lambda = np$

λ is the rate of occurrence of the rare events

- The number of misprints on a page(or a group of pages) of a book
- The number of people in a community who survive to age 100
- The number of wrong telephone numbers that are dialed in a day
- The number of earthquakes occurring during some fixed timespan

Example

Suppose that the number of typographical errors on a single page of this book has a Poisson distribution with parameter $\lambda = 1/2$. Calculate the probability that there is at least one error on this page.
If X is the number of errors on this page, then

$$P(X \geq 1) = 1 - P(X = 0) = 1 - e^{-1/2} = 0.393$$

The mean number of errors on the page $E(X) = 1/2$.

Example

The Poisson random variable may be used as an approximation for a binomial random variable with parameters (n, p) when n is large and p is small enough so that np is of moderate size
ex. Suppose that the probability that an item produced by a certain machine will be defective is 0.1. Find the probability that a sample of 10 items will contain at most 1 defective item.

Example

The owner of a certain website is studying the distribution of the number of visitors to the site. Every day, a million people independently decide whether to visit the site, with probability $2 \cdot 10^{-6}$ of visiting. Give a good approximation for the probability of getting at least three visitors on a particular day. Let the number of visitors be 10^6.

Example

A toll bridge charges \$1 for motorcycles and \$2.50 for other vehicles. Suppose that during daytime hours, 10% of all vehicles are motorcycles. If 250 vehicles cross the bridge during a particular daytime period, what is the resulting expected toll revenue? What is the variance?

Let X be the number of motorcycles and Y is the revenue. Find E(Y), Var(Y).