Binomial distribution

Anastasiia Kim

February 14, 2020

Example

There are n eggs, each of which hatches a chick with probability p (independently). Each of these chicks survives with probability r, independently.

- What is the distribution of the number (H) of chicks that hatch?
- What is the expected number of chicks that hatch?
- What is the distribution of the number (S) of chicks that survive?

R: Binomial distribuion

Useful functions in R: *dbinom*, *pbinom*, *rbinom*:

- *dbinom* is the Binomial pmf p(x). It takes three inputs: the first is the value of x at which to evaluate the pmf, and the second and third are the parameters n and p. For example, $dbinom(3, 5, 0.2)$ returns the probability $P(X = 3)$ where $X \sim Bin(5, 0.2)$

$$dbinom(3, 5, 0.2) = \binom{5}{3}(0.2)^3(0.8)^2 = 0.0512$$

R: Binomial distribuion

- *pbinom* is the Binomial cdf F(x). It takes three inputs: the first is the value of x at which to evaluate the CDF, and the second and third are the parameters. For example, $pbinom(3, 5, 0.2)$ is the probability $P(X \leq 3)$ where $X \sim Bin(5, 0.2)$

$$pbinom(3, 5, 0.2) = \sum_{k=0}^{3} \binom{5}{k} (0.2)^k (0.8)^{5-k} = 0.9933$$

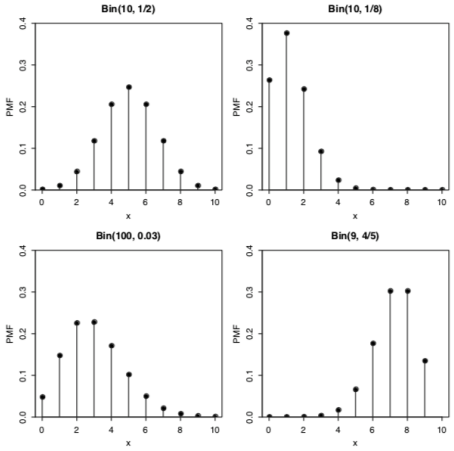
R: Binomial distribuion

- *rbinom* is a function for generating Binomial random variables. For *rbinom*, the first input is how many r.v.s we want to generate, and the second and third inputs are still the parameters. For example, $rbinom(3, 5, 0.2)$ produces realizations of three i.i.d. Bin(5, 0.2) r.v.s

$$2 \quad 1 \quad 1$$

R: Binomial distribuion

plot(0:10, dbinom(0:10, 10, 0.5), type = 'h', xlab = 'x', ylab = 'p(x)', main = 'pmf for a Binomial(n = 10, p = 0.5) r.v.')



Example

Each sample of water has a 10% chance of containing a particular organic pollutant. Assume that the samples are independent with regard to the presence of the pollutant. Let X = the number of samples that contain the pollutant in the next 18 samples analyzed.

- Find the probability that in the next 18 samples, exactly 2 contain the pollutant
- Determine the probability that at least four samples contain the pollutant
- Find $P(3 \leq X < 7)$

Example

A new treatment for a disease is being tested, to see whether it is better than the standard treatment. The existing treatment is effective on 50% of patients. It is believed initially that there is a $2/3$ chance that the new treatment is effective on 60% of patients, and a $1/3$ chance that the new treatment is effective on 50% of patients. In a pilot study, the new treatment is given to 20 random patients, and is effective for 15 of them. Given this information, what is the probability that the new treatment is better than the standard treatment ($P(B|X = 15)$)?