Homework 4. Part one

Stat 345 - Spring 2020

Name: ____

Problem 1

The maximum time to complete a task in a project is 2.5 days. Suppose that the completion time as a proportion of this maximum is a Beta random variable with mean 0.4 and variance 0.2.

a) Write the probability distribution function (pdf).

b) What is the probability that the task requires more than two days to complete?

Problem 2

The number of cracks in a section of interstate highway that are significant enough to require repair is assumed to follow a Poisson process with rate $\lambda = 2$ cracks per one hour drive. What is the probability that I have to drive at least 3 hours before seeing the first crack?

Problem 3

The survival function is $S(t) = 1 - F(t)$, or the probability that a person/machine/business lasts longer than t time units. The hazard function is $h(t) = f(t)/S(t)$. Here $F(t)$ is the cdf and $f(t)$ is the pdf. It is the probability that the person/machine/business dies in the next instant, given that it survived to time t. Determine the hazard function for the Exponential(λ) distribution. How does the expression for the exponential hazard function related to the memoryless property of the exponential distribution (explain in words)?

Problem 4

Suppose that light bulbs have a failure time that is exponentially distributed, with mean survival time 3 months.

a) Suppose $n = 5$ bulbs used simultaneously. What is the probability at most one bulb burns out in the first six months? (*Hint:* first find the probability that the bulb will survive six months. Then model the number of burnt out bulbs with a certain discrete distribution.)

b) Let Y_1, Y_2, Y_3, Y_4 be the length of time each bulb survives. What is the distribution of $Y_1 + Y_2 + Y_3 + Y_4$?