CHAPTER 12: Introducing Probability

Basic Practice of Statistics
7th Edition

Lecture PowerPoint Slides

In Chapter 12, we cover …

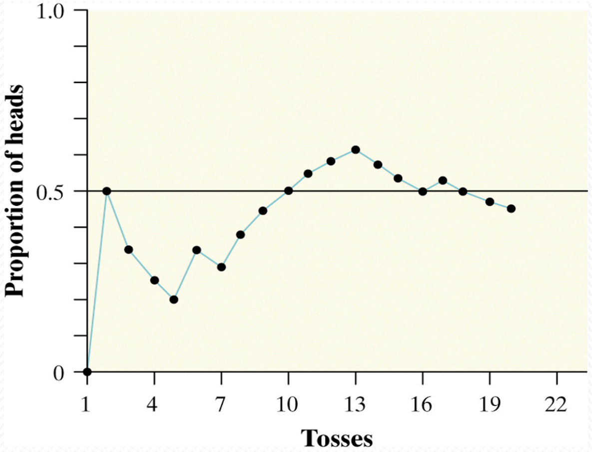
- The idea of probability
- The search for randomness*
- Probability models
- Probability rules
- Finite and discrete probability models
- Continuous probability models
- Random variables
- Personal probability*

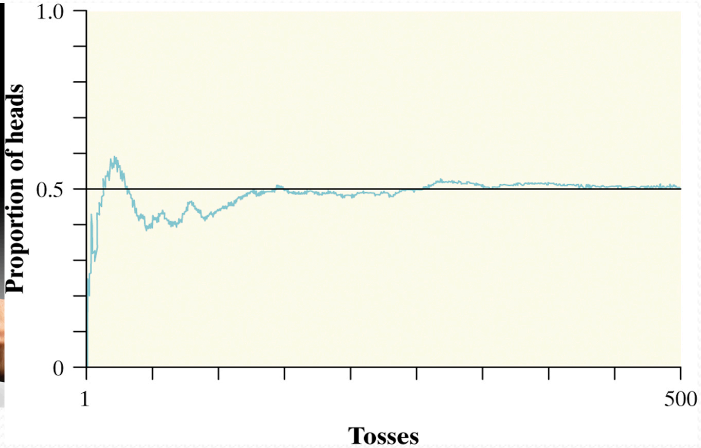
The idea of probability

Chance behavior is unpredictable in the short run, but has a regular and predictable pattern in the long run.

RANDOMNESS AND PROBABILITY

- We call a phenomenon random if individual outcomes are uncertain but there is nonetheless a regular distribution of outcomes in a large number of repetitions.
- The probability of any outcome of a random phenomenon is the proportion of times the outcome would occur in a very long series of repetitions.





Search for randomness*

- Computer programs, including applets
- Random number table
- Physical means, such as tossing coins or rolling dice
- “Chaos theory”
- Quantum mechanics

Probability models

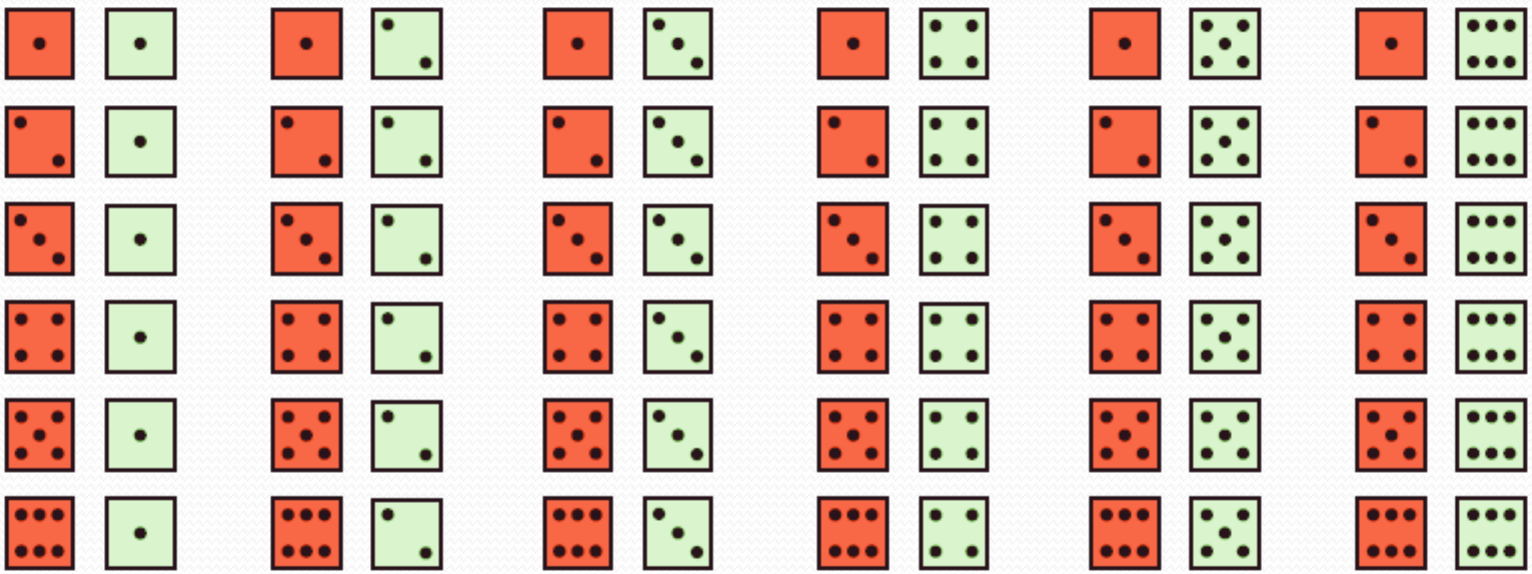
- Descriptions of chance behavior contain two parts: a list of possible outcomes and a probability for each outcome.

PROBABILITY MODELS

- The sample space S of a random phenomenon is the set of all possible outcomes.
- An event is an outcome or a set of outcomes of a random phenomenon. That is, an event is a subset of the sample space.
- A probability model is a mathematical description of a random phenomenon consisting of two parts: a sample space S and a way of assigning probabilities to events.

Probability models

Example: Give a probability model for the chance process of rolling two fair, six-sided dice—one that's red and one that's green.



Sample Space
36 Outcomes

Since the dice are fair, each outcome is equally likely.
Each outcome has probability 1/36.

Probability rules

1. **Any probability is a number between 0 and 1.** Any proportion is a number between 0 and 1, so any probability is also a number between 0 and 1.

2. **All possible outcomes together must have probability 1.** Because some outcome must occur on every trial, the sum of the probabilities for all possible outcomes must be exactly 1.

3. ***If two events have no outcomes in common*, the probability that one or the other occurs is the sum of their individual probabilities.**

4. **The probability that an event does not occur is 1 minus the probability that the event does occur.** The probability that an event occurs and the probability that it does not occur always add to 100%, or 1.

Probability rules

The probability rules in formal language:

PROBABILITY RULES

Rule 1. The probability $P(A)$ of any event *A* satisfies $\mathbf{0 \leq P(A) \leq 1}$.

Rule 2. If *S* is the sample space in a probability model, then $\boldsymbol{P(S) = 1}$.

Rule 3. Two events A and B are disjoint if they have no outcomes in common and so can never occur together. If *A* and *B* are disjoint, ***P*(*A* or *B*) = *P*(*A*) + *P*(*B*)**.

This is the **addition rule for disjoint events.**

Rule 4: For any event *A*, ***P*(*A* does not occur) = 1 − *P*(*A*).**

Probability rules—example

First digits of numbers in legitimate financial records often follow a model known as Benford's law. Call the first digit of a randomly chosen record X—Benford's law gives this probability model for X:

First digit, X:	1	2	3	4	5	6	7	8	9
Probability	0.301	0.176	0.125	0.097	0.079	0.067	0.058	0.051	0.046

(a) Show that this is a legitimate probability model.

Each probability is between 0 and 1 and
0.301 + 0.176 + … + 0.046 = 1

(b) Find the probability that the first digit for the chosen number is not a 9.

***P*(not 9) = 1 – *P*(9)**
= 1 – 0.046 = 0.954

Finite and discrete probability models

- One way to assign probabilities to events is to assign a probability to every individual outcome, then add these probabilities to find the probability of any event. This idea works well when there are only a finite (fixed and limited) number of outcomes.

FINITE PROBABILITY MODEL

- A probability model with a finite sample space is called finite.
- To assign probabilities in a finite model, list the probabilities of all the individual outcomes. These probabilities must be numbers between 0 and 1 that add to exactly 1. The probability of any event is the sum of the probabilities of the outcomes making up the event.

- Discrete probability models include finite models as well as sample spaces that are infinite and equivalent to the set of all positive integers.
- The Benford's law probability model was *finite*.

Continuous probability models

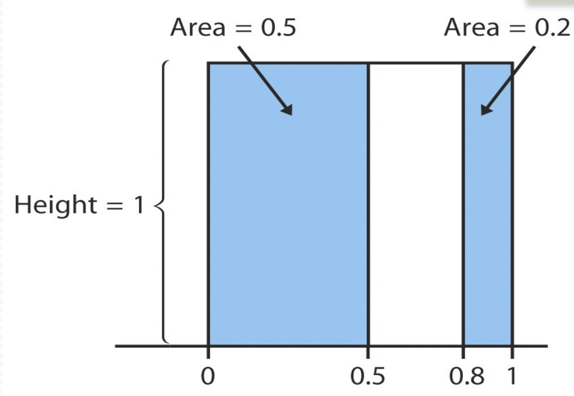
- Suppose we want to choose a number at random between 0 and 1, allowing any number between 0 and 1 as the outcome.
- We cannot assign probabilities to each individual value because there is an infinite interval of possible values.

CONTINUOUS PROBABILITY MODEL

- A continuous probability model assigns probabilities as areas under a density curve. The area under the curve and above any range of values is the probability of an outcome in that range.

- Example: Find the probability of getting a random number that is less than or equal to 0.5 OR greater than 0.8.

Uniform Distribution



Normal probability models

- We can use any density curve to assign probabilities. The density curves that are most familiar to us are the Normal curves.
- Normal distributions are continuous probability models as well as descriptions of data.
- For example, if we look at the heights of all young women, we find that they closely follow the Normal distribution with mean $\mu = 64.3$ inches and standard deviation $\sigma = 2.7$ inches. Call her height X. If we repeat the random choice very many times, the distribution of values of X is the same Normal distribution that describes the heights of all young women.
- Like all continuous probability models, the normal assigns probability **0** to every **individual outcome**.
- The technique for finding such probabilities is found in Chapter 3.

Random variables

RANDOM VARIABLE

- A random variable is a variable whose value is a numerical outcome of a random phenomenon.
- The probability distribution of a random variable *X* tells us what values *X* can take and how to assign probabilities to those values.

- Random variables that have a finite list of possible outcomes are called discrete.
- Random variables that can take on any value in an interval, with probabilities given as areas under a density curve, are called continuous.