Tests of significance

Chapter 17

Hypotheses are always stated in terms of population parameters.
H_0 is a null hypothesis. H_0 is often a statement that no effect or no difference is present.
H_a is an alternative hypothesis. H_a says that a parameter differs from its null value μ_0 in a specific direction (one-sided alternative) or in either direction (two-sided alternative).
Significance tests for the null hypothesis $H_0 : \mu = \mu_0$ concerning the unknown population (true) mean μ are based on the **one-sample z test statistic**:

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

The z test assumes an SRS of size n from the population with Normal distribution.
Notation $P(Z \leq z)$=$P(Z < z)$ means the probability that variable Z is less than value of the test statistic z.

H_0 :	$\mu = \mu_0$	$\mu = \mu_0$	$\mu = \mu_0$
H_a :	$\mu < \mu_0$	$\mu > \mu_0$	$\mu \neq \mu_0$
$P - value$	$P(Z \leq z)$	$P(Z \geq z)$	$2P(Z \geq \lvert z \rvert)$
Table A	area to the left of z	area to the right of z	2 (area to the right of $\lvert z \rvert$)

Small P-value indicate strong evidence against H_0. If the P-value is as small or smaller than a specified value α, the data are statistically significant at significance level α.

1. Dementia is the loss of the intellectual and social abilities severe enough to interfere with judgment, behavior, and daily functioning. Alzheimer's disease is the most common type of dementia. One article explored the experience and struggles of people diagnosed with dementia and their families. For a SRS of 21 people with early-onset dementia the mean age at diagnosis was 52.5 years. At the 5% significance level, do the data provide sufficient evidence to conclude that the mean age at diagnosis of all people with early-onset dementia is less than 55 years old? Assume that the population has a normal distribution with standard deviation of 6.8 years.
a) State your hypotheses in words.

b) Calculate the test statistic and find the p-value.

c) Write your conclusion in terms of the problem.

2. Statistics can help decide the authorship of literary works. Sonnets by a certain Elizabethan poet are known to contain an average of 8.9 new words (words not used in the poets other works). The standard deviation of the number of new words is $\sigma = 2.5$. Now a manuscript with 6 new sonnets has come to light, and scholars are debating whether it is the poets work. The new sonnets contain an average of 10.2 words not used in the poets known works. We expect poems by another author to contain more new words. At the 1% significance level, is there evidence that the new sonnets are not by our poet?